Licenciatura Management

Operational Research Chapter 1

2018-2019

1911-2011 ANOS

100 ANOS A PENSAR NO FUTURO

Teacher

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Bibliography

- F.S. Hillier, G.J. Lieberman, *Introduction to Operations Research*, 9th edition, McGraw-Hill, International Edition, New York, 2010;
- M.C. Mourão, L. Santiago Pinto, O. Simões, J. Valente, M.V. Pato, *Investigação Operacional: Exercícios e Aplicações*, 1ª edição, Verlag Dashöfer, Lisboa, 2011 (in portuguese)

Assessment Process

- During the break week there will be a midterm exam that covers the first half of the program. In the "época Normal" (EN) there will be a second midterm that covers the second half of the program. In this day students who wish may choose to repeat the evaluation of the first half of the program. Each half of the program has a 50% worth of the final grade .
- Students who do not get a grade higher than or equal to 9.5 in the average of the two mini-tests or examination of EN, or a minimum of (8 in 20, in each mid term) will be submitted to the "época de Recurso" .
- Students who obtain 17.5 or more and would like to have a final grade greater than 17 may be called to an oral exam.
- It is allowed to consult 1 A4 sheet in the midterms and 2 A4 sheets in the other exams .
- Calculating machines are not allowed, neither in the exams nor in the midterms.
- All that is not specified above follows the "Regime Geral de Avaliação de Conhecimentos".

Links

INFORMS - Institute of Operations Research and MAnagement Sciences

https://www.informs.org/

APDIO - Associação Portuguesa de Investigação Operacional (portuguese association)

http://apdio.pt/home

Investigação Operacional. InternetLink

"Operations Research. Models and Methods. Internet", Paul O. Jensen

www.me.utexas.edu/~jensen/ ORMM

Frederick S. Hillier

https://engineering.stanford.edu/people/frederick-s-hillier

Gerald J. Lieberman 31/12/1925 -18/5/1999

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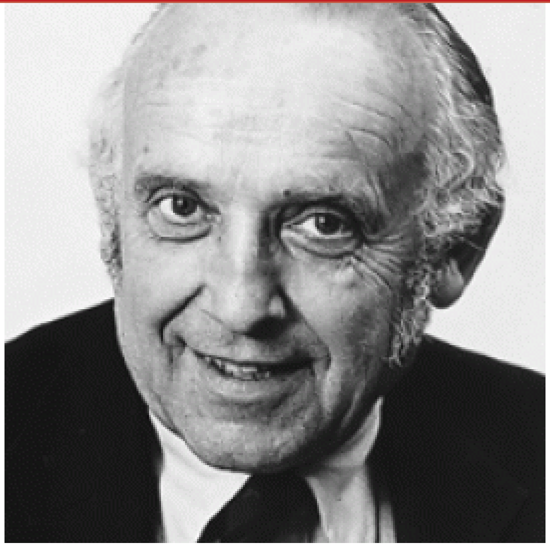


ORMS By Vijay Mehrotra

And then one day Tom Cook come into town to give a department seminar. Dr. Cook was already one of my heroes, an academic turned management consultant turned airline industry executive/management science champion. When he was asked about what training he thought was most important for success in industry, his answer was both direct and surprising

“Give me someone who really understands the Hillier and Lieberman book and who can communicate well with people, and I can almost guarantee you that they will make a big contribution to our company," said the head of operations research at American Airlines. But no bigger than the one that Jerry himself made to our profession.

In honor of you, Jerry, I'm taking the rest of the day off. Reference: 1. 1997, Kimball Medal citation.

"https://www.informs.org/ORMS-Today/Archived-Issues/orms-8-99/Academic-Descendant-of-Prof.-Lieberman

TABLE 1.1 Applications of operations research to be described in application vignettes

Organization	Area of Application	Section	Annual Savings
Federal Express	Logistical planning of shipments	1.3	Not estimated
Continental Airlines	Reassign crews to flights when schedule disruptions occur	2.2	$40 million
Swift & Company	Improve sales and manufacturing performance	3.1	$12 million
Memorial Sloan-Kettering Cancer Center	Design of radiation therapy	3.4	$459 million
United Airlines	Plan employee work schedules at airports and reservations offices	3.4	$6 million
Welch's	Optimize use and movement of raw materials	3.3	$150,000
Samsung Electronics	Reduce manufacturing times and inventory levels	4.3	$200 million more revenue
Pacific Lumber Company	Long-term forest ecosystem management	6.7	$398 million NPV
Procter & Gamble	Redesign the production and distribution system	8.1	$200 million
Canadian Pacific Railway	Plan routing of rail freight	9.3	$100 million
United Airlines	Reassign airplanes to flights when disruptions occur	9.6	Not estimated
U.S. Military	Logistical planning of Operations Desert Storm	10.3	Not estimated
Air New Zealand	Airline crew scheduling	11.2	$6.7 million
Taco Bell	Plan employee work schedules at restaurants	11.5	$13 million
Waste Management	Develop a route-management system for trash collection and disposal	11.7	$100 million

TABLE 1.1 Applications of operations research to be described in application vignettes

Organization	Area of Application	Section	Annual Savings
Bank Hapoalim Group	Develop a decision-support system for investment advisors	12.1	$31 million more revenue
Sears	Vehicle routing and scheduling for home services and deliveries	13.2	$42 million
Conoco-Phillips	Evaluate petroleum exploration projects	15.2	Not estimated
Workers' Compensation Board	Manage high-risk disability claims and rehabilitation	15.3	$4 million
Westinghouse	Evaluate research-and-development projects	15.4	Not estimated
Merrill Lynch	Manage liquidity risk for revolving credit lines	16.2	$4 billion more liquidity
PSA Peugeot Citroën	Guide the design process for efficient car assembly plants	16.8	$130 million more profit
KeyCorp	Improve efficiency of bank teller service	17.6	$20 million
General Motors	Improve efficiency of production lines	17.9	$90 million
Deere & Company	Management of inventories throughout a supply chain	18.5	$1 billion less inventory
Time Inc.	Management of distribution channels for magazines	18.7	$3.5 million more profit
Bank One Corporation	Management of credit lines and interest rates for credit cards	19.2	$75 million more profit
Merrill Lynch	Pricing analysis for providing financial services	20.2	$50 million more revenue
AT&T	Design and operation of call centers	20.5	$750 million more profit

Course contents:

1. Linear Programming
2. The Simplex Method
3. Duality and Sensitivity Analysis
4. The Transportation and the Assignment Problems
5. Network Optimization
6. Integer Linear Programming

Objectives of the course:

The objective of this course is to introduce the students to the wide field of applications for (integer) linear programming and network models, as well as provide basic knowledge of the respective mathematical models.

Students will be required to apply very simple algorithms and dominate the resolution of (integer) linear programming problems with the Solver/Excel software. Special emphasis will be given to the economic interpretation of results.

Detailed program:

1. Linear Programming (LP)

1.1 Introduction

1.2 Formulation and Graphical Solution

1.3 Definitions and Properties

1.4 Solving Problems by Solver/Excel

2. Simplex Method

2.1 Introduction

2.2 Augmented Form and Basic Feasible Solutions

2.3 Simplex Algorithm

3. Duality and Sensitivity Analysis

3.1 Introduction

3.2 Duality

3.3 Economic Interpretation of Duality. Shadow Prices. Primal-Dual Relations

3.4 Sensitivity Analysis

- Changes in the Right-Hand Sides of the Constraints
- Changes in the Coefficients of the Objective Function

4. Transportation and Assignment Problems

4.1 Introduction

4.2 Transportation Problem

4.3 Assignment Problem

5. Network Optimization

5.1 Introduction

5.2 Minimum Cost Flow Problem

5.3 Shortest-Path Problem

5.4 Minimum Spanning Tree Problem

- Prim Algorithm

6. Integer Linear Programming (ILP)

6.1 Introduction

6.2 Integer Linear Programming Problems

6.3 Graphical and Solver/Excel Solution

6.4 Formulations with Binary Variables

1. Linear Programming (LP)

1.1 Introduction

1.2 Formulation and Graphical Solution

1.3 Definitions and Properties

1.4 Solving Problems by Solver/Excel

Prototype Example 1

x_1 – no. batches of P1 produced per week (P1=8-foot glass door with aluminum framing)

x_2 – no. batches of P2 produced per week (P2=4×6 foot double-hung wood framed window)

Z – total profit per week (in thousands of dollars) from producing these two products

Linear Programming (LP) Model:

$$Max\ Z = 3x_1 + 5x_2$$

$$s.t. \begin{cases} x_1 \le 4 \\ 2x_2 \le 12 \\ 3x_1 + 2x_2 \le 18 \\ x_1, x_2 \ge 0 \end{cases}$$

LP model – standard form

$$Z^* = Max\ Z = \sum_{j=1}^{n} c_j x_j$$ Objective Function (OF)

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \leq b_i & i = 1, 2, \dots, m \\ x_j \geq 0 & j = 1, 2, \dots, n \end{cases}$$

Functional Constraints
Sign Constraints

Decision variable: x_j $(j = 1, \dots, n)$ represents level of activity *j*

Data:

c_j coefficient on the objective function of the decision variable *j*;

b_i righ-hand-side (RHS) of the functional constraint *i*;

a_{ij} technical coefficient of the decision variable *j* on the functional constraint *i.*

c_j,b_i and a_{ij} are called the parameters of the LP model

Linear Programming

Assumptions of Linear Programming

Proportionality: The contribution of each activity (j) to the value of the objective function and to the left-hand-side of the constraints is proportional to the level of the activity (x_j).

Additivity: The value of the objective function and the value of the left-hand-side of the constraints are the sum of the individual contributions of the various activities.

Divisibility: The variables assume real values ($x_j \in R$).

Certainty: Every coefficient (also called parameter) is assumed to be a known constant.

Linear Programming

Definitions I

Solution of an LP – a vector of R^n which components are the values of the variables;

Feasible Solution (FS) – a solution that satisfies all the constraints (functional and sign);

Non Feasible Solution (NFS) – a solution that does not satisfy at least one of the constraints;

Feasible Region (FR) – the set of all feasible solutions;

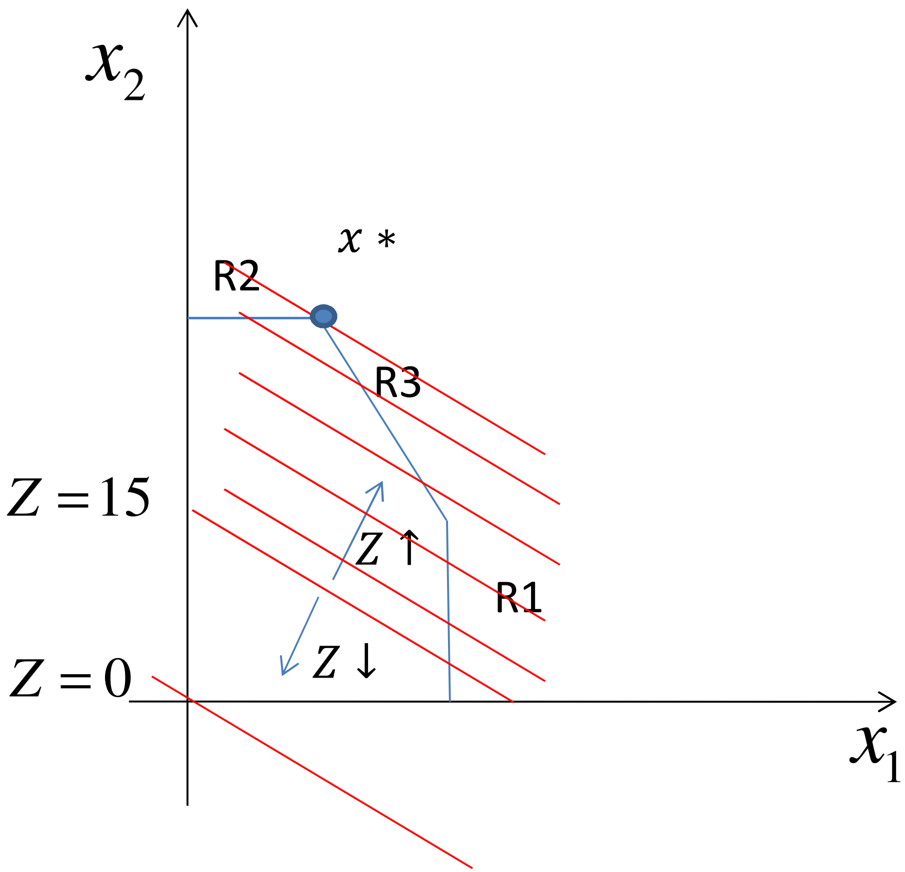
Optimal Solution (OS) – a feasible solution that gives the best value to the objective function (OF) (the best value=maximum or minimum);

Optimal value – the value of the objective function at an optimal solution;

Binding constraint in a solution – a constraint that hold with equality at that solution;

To solve an LP is to determine the optimal solution (or solutions) and the optimal value or to conclude that an optimal solution does not exist and why.

Graphical Method – Prototype Example 1



$$Max\ z = 3x_1 + 5x_2$$

$$\begin{cases} x_1 \leq 4 & \text{(R1)} \\ 2x_2 \leq 12 & \text{(R2)} \\ 3x_1 + 2x_2 \leq 18 & \text{(R3)} \\ x_1,\ x_2 \geq 0 \end{cases}$$

$$\begin{cases} 2x_2 = 12 \\ 3x_1 + 2x_2 = 18 \end{cases} \cdots \begin{cases} x^*_1 = 2 \\ x^*_2 = 6 \end{cases}$$

$$Z^* = 36$$

Linear Programming

Graphical Method

1) Represent (in the $\boldsymbol{x_1}$-$\boldsymbol{x_2}$ plan) the FR = intersection of the half-plans defined by the constraints of the LP (functional and sign constraints);

2) If FR = { } the problem is infeasible. STOP.

3) Otherwise (FR ≠ { }) identify the optimal solution (or solutions), if any.
   - Set Z to K (arbitrarily fixed) and represent the line $c_1x_1 + c_2x_2 = K$. Identify the half-plan conducting to better values of Z. Then
   - Identify the optimal solution as the point in the FR with the best value of Z (it can be more than one), or
   - conclude that the problem is unbounded, (there is no optimal solution).

a) $Max\ z = x_1 + 2x_2$

$$\text{s. t.} \begin{cases} x_1 - 2x_2 \leq 3 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

b) $Max\ z = 3x_1 + 4x_2$

$$\text{s. t.} \begin{cases} x_1 - 2x_2 \geq 4 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

c) $Max\ z = x_1 + x_2$

$$\text{s. t.} \begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$

d) $Max\ z = x_1 - x_2$

$$\text{s. t.} \begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$

e) $Max\ z = -10x_1 - 5x_2$

$$\text{s. t.} \begin{cases} x_1 - x_2 \leq 5 \\ x_1 + \frac{8}{5}x_2 \geq -3 \\ x_1 \text{free} \\ x_2 \leq 0 \end{cases}$$

h) $min\ z = x_1 + x_2$

$$\text{s. t.} \begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq -2 \\ x_1, x_2 \geq 0 \end{cases}$$

j) $Max\ z = 3x_1 + 6x_2$

$$\text{s. t.} \begin{cases} x_1 + 2x_2 \leq 4 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$

LP – solving by solver of excel – prototype example 1

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data

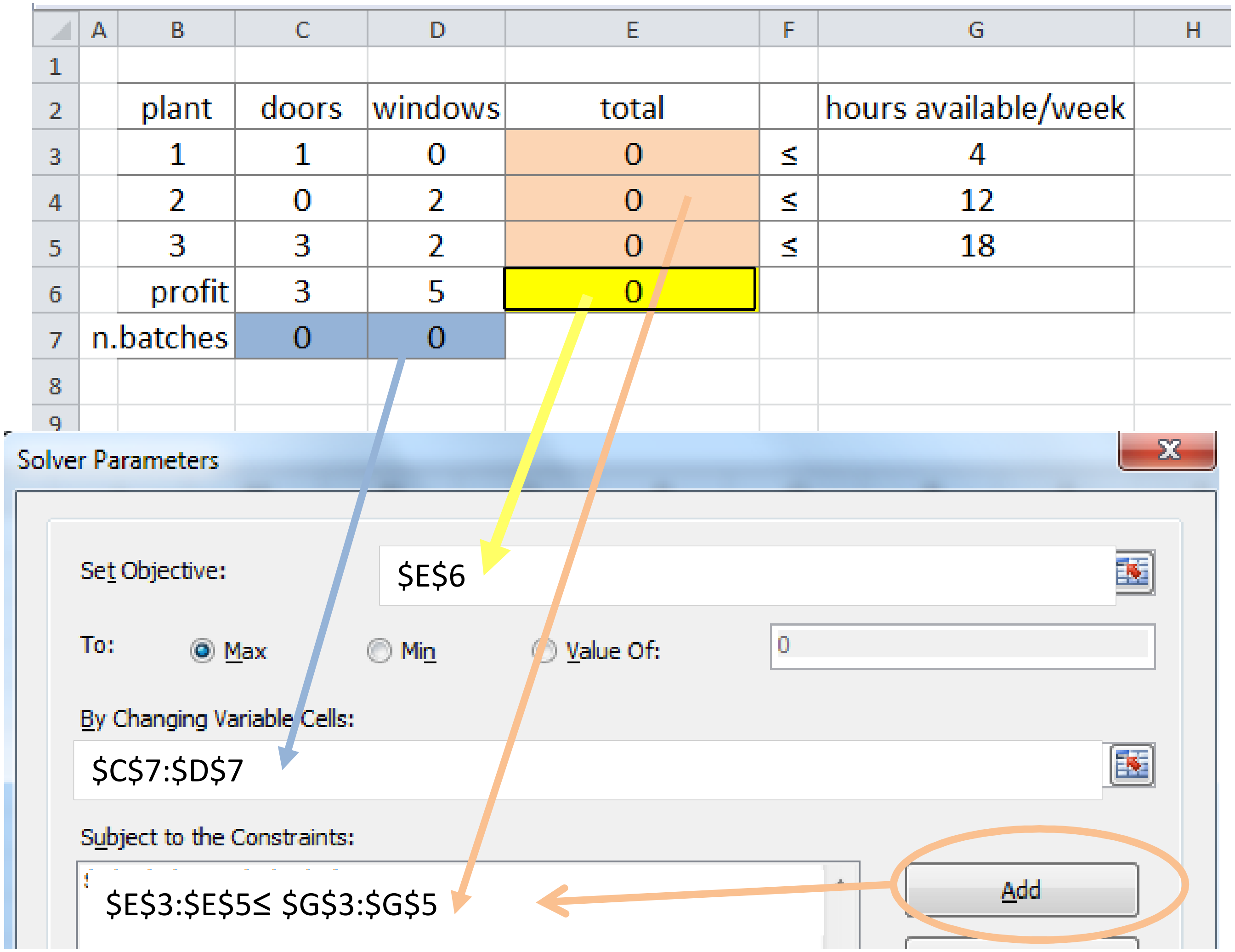
data

	A	B	C	D	E	F	G	H
1								
2		plant	doors	windows	total		hours available/week	
3		1	1	0	0	≤	4	
4		2	0	2	0	≤	12	
5		3	3	2	0	≤	18	
6		profit	3	5	0			
7	n.batches		0	0				
8								
9								

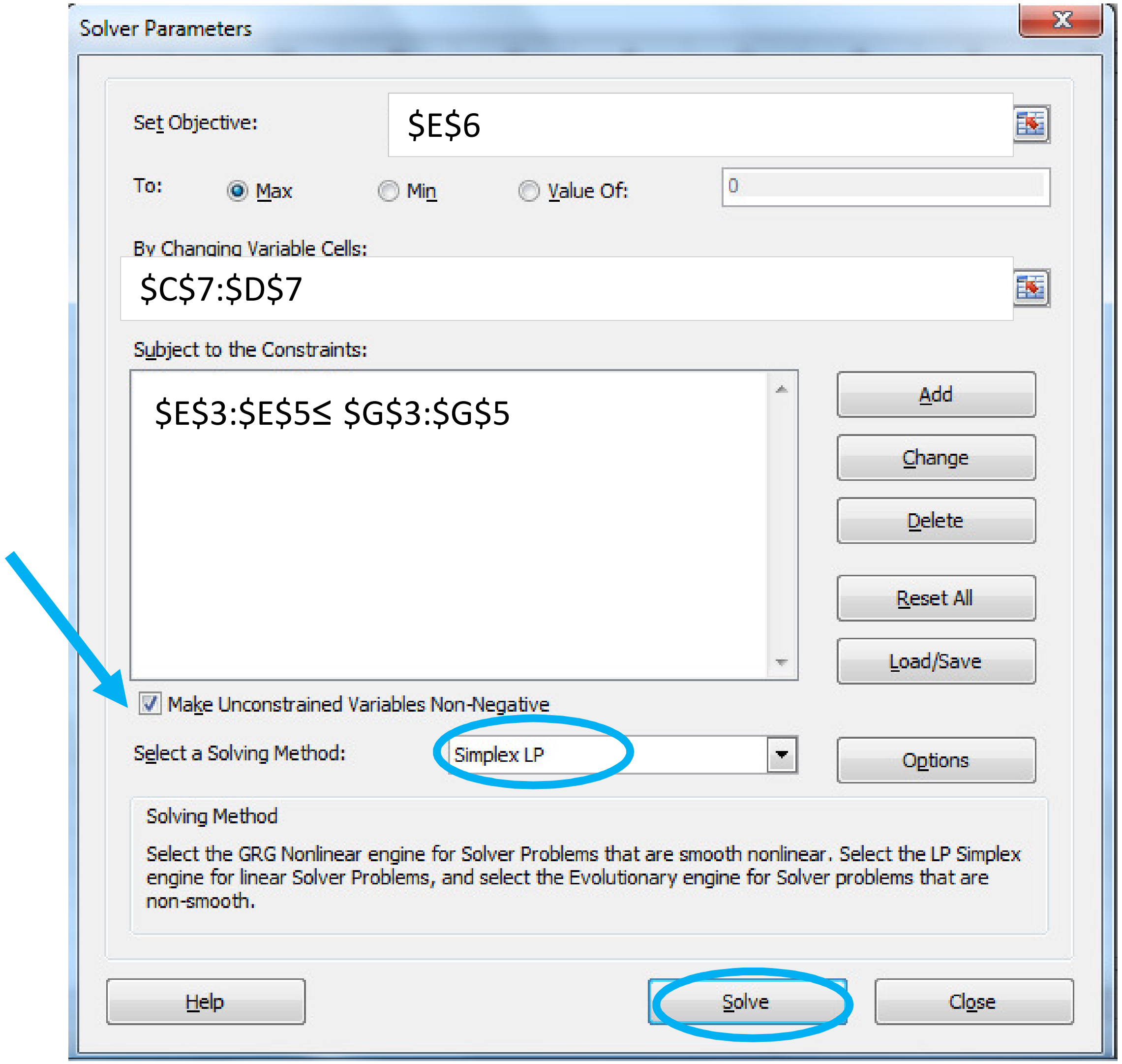
initial values

	E
1	
2	total
3	=SUMPRODUCT(C3:D3;$C$7:$D$7)
4	=SUMPRODUCT(C4:D4;$C$7:$D$7)
5	=SUMPRODUCT(C5:D5;$C$7:$D$7)
6	=SUMPRODUCT(C6:D6;$C$7:$D$7)
7	

LP – solving by solver of excel – prototype example 1



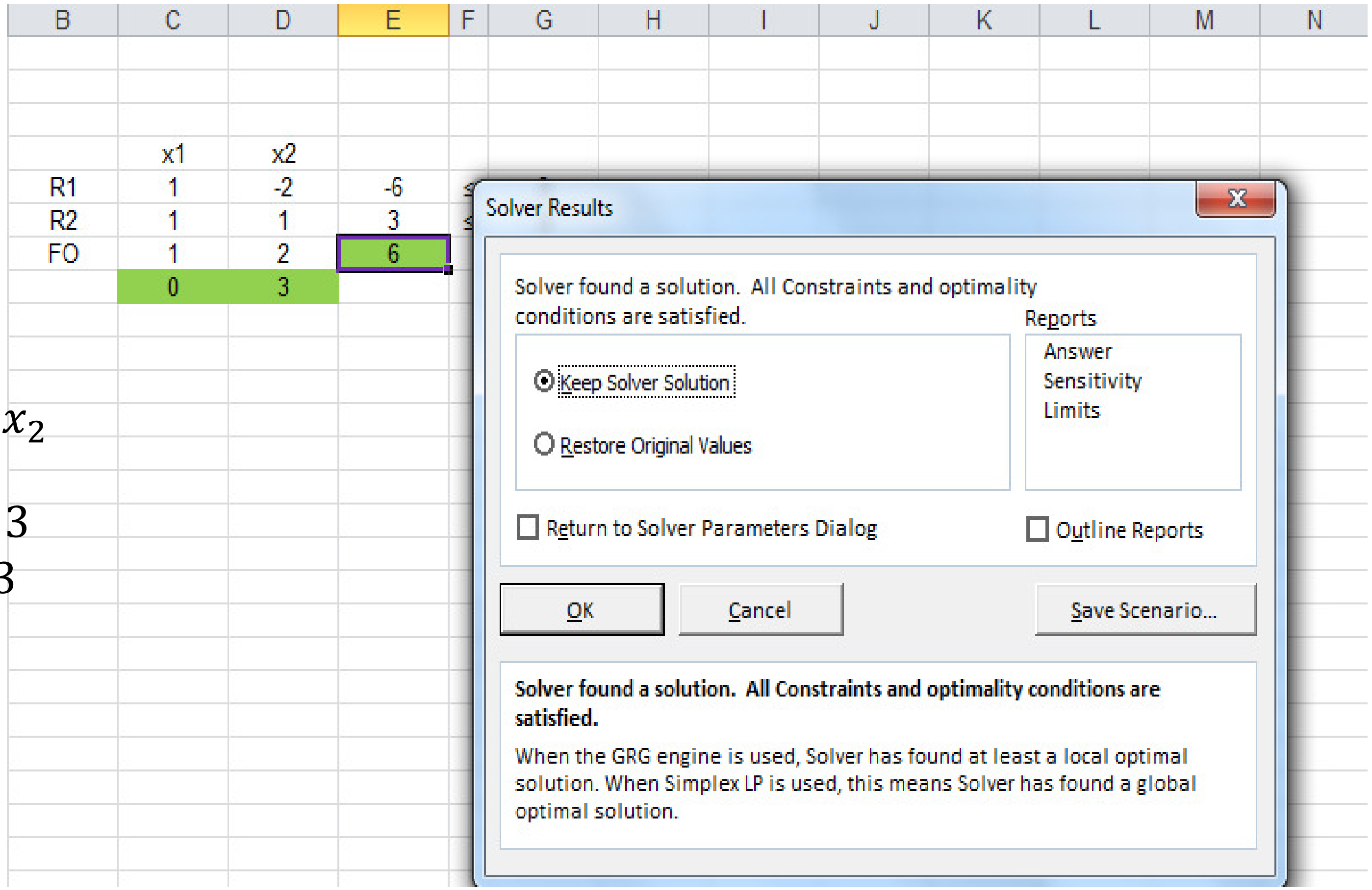
LP – solving by solver of excel – prototype example 1



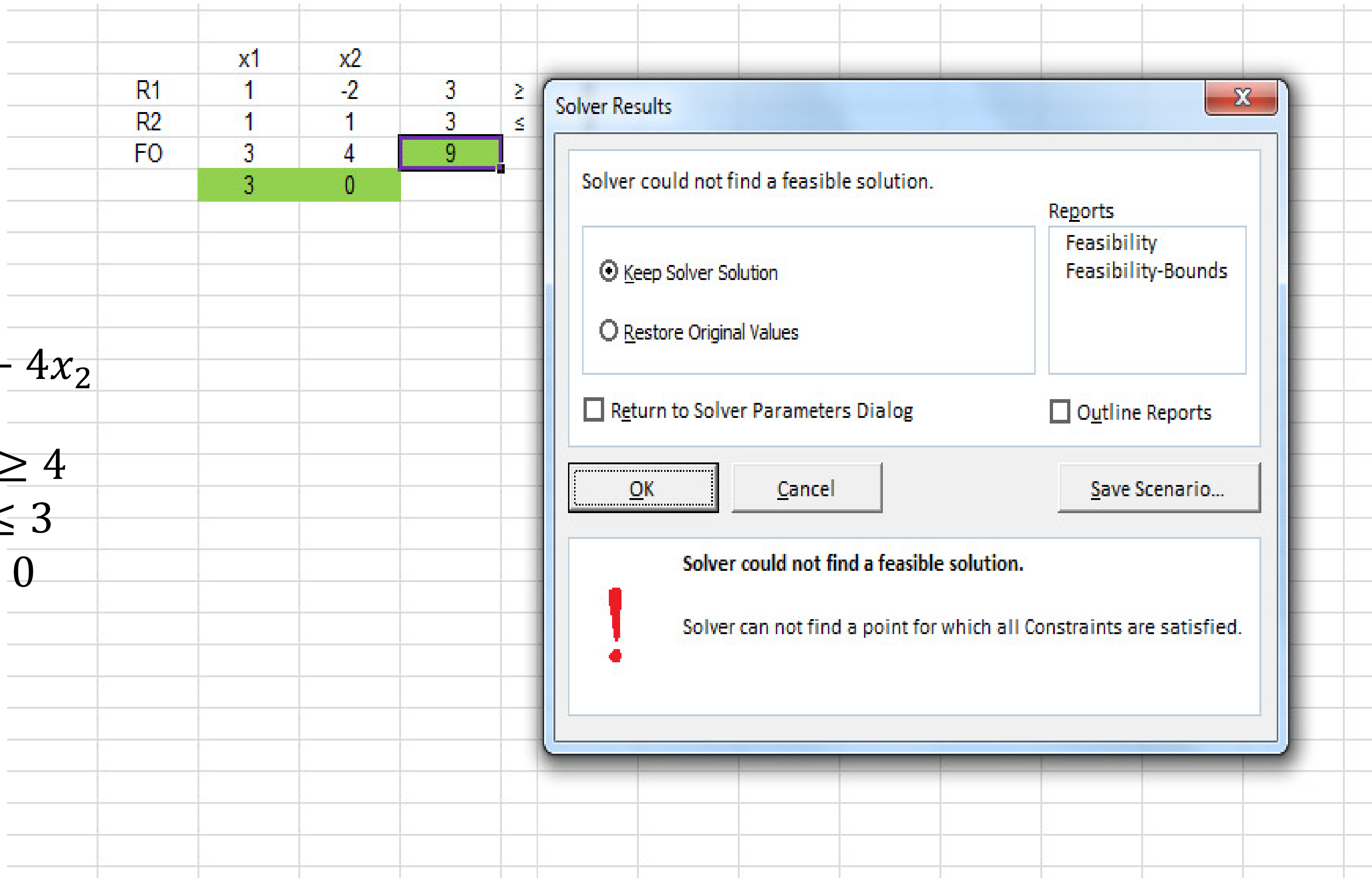
1 a)

$$Max\ z = x_1 + 2x_2$$

$$\text{s. t.}\begin{cases} x_1 - 2x_2 \leq 3 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$



1 b) infeasible



$$Max\ z = 3x_1 + 4x_2$$

$$s.t. \begin{cases} x_1 - 2x_2 \geq 4 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

1 c) unbounded

	x1	x2			
R1	1	-1	2	≤	2
R2	1	-1	2	≥	0
FO	1	1	2		
	2	0			

$$Max\ z = x_1 + x_2$$

$$\text{s. t.} \begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$